Complex optimization problems in space systems

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Introduction

Space systems are characterized by the huge cost induced by their design, their launch, and their maintenance. Furthermore, these systems have to satisfy more and more difficult criteria of performance or quality of service. Thus, the management of resources involved in these systems leads to specific and complex optimization problems [Hall 94, Frank 01]. There are two main kinds of problems: 1) planning and assignment problems of communication slots between ground station and satellites; 2) problems of action planning on board a spacecraft.

In this paper, we address problems stemming from two different space mission projects of the CNES (French Space Agency). The first one is the Netlander mission which aims at a scientific investigation of Mars with help of probes on its surface. In the context of this mission, we are interested in both planning communications periods between Mars probes and satellites orbiting Mars, and planning experiments on probes. Linear programming and constraint propagation are developed.

The second problem is in the context of the Pleiades program of Earth observing with satellites of new generation. The problem consists in selecting and scheduling images taken by one satellite in order to maximize a quality criterion depending on the acquisition rate of an image. We propose a generalized linear programming (column generation) approach for this problem.

Netlander mission

Problem description

Netlander mission consists in deploying and operating a network of 4 probes on the surface of Mars to perform geophysical and atmospheric investigations of the planet. During the mission, the probes communicate with the Earth via satellites orbiting Mars. These communications are used to download to the Earth data of acquired experiments results on probes, and to upload new workplans (planning of experiments) from the Earth to each probe.

The decision problem is both planning probes/orbiters communication slots and planning tasks composing the workplans uploaded to the probes. The expected solutions have to satisfy strong resource and temporal constraints. Moreover, several quality criteria have to be optimised (speed of data download, quantity and relevance of experiments...).

We propose to decompose this complex decision problem into two subproblems. First we consider the communication slots planning problem; we model and solve it with linear programming. Then, we propose a constraint based approach for the experiments planning problem.

These two subproblems are linked by the energy resource of the probes, used both to achieve experiments and to communicate with orbiters. However the decomposition can be easily justified since the two subproblems arise at different stages of the project.

Communications planning

This first subproblem is well defined in terms of constraints and objective function. Time horizon T is discretized in periods (typically of 120 seconds) ; the decision variables are $X_{s,t}$ equal to 1 if the probe s communicates with the satellite at period t, 0 otherwise. The problem can be formulated as follows:

$$\max \sum_{s=1}^{4} \sum_{t=1}^{T} X_{s,t} \tag{1}$$

$$X_{s,t} = 0 \qquad {\scriptstyle s=1..4,\ t\in I_s} \tag{2}$$

$$\sum_{s=1}^{4} X_{s,t} \leq 1 \qquad {\scriptstyle t=1..T} \tag{3}$$

$$\sum_{t'=t+1}^{t+d_{min}} X_{s,t'} \geq d_{min}(X_{s,t+1} - X_{s,t}) \qquad {\scriptstyle t=1..(T-d_{min}),\ s=1..4} \tag{4}$$

$$X_{s,t} \leq X_{s,t-1} \qquad {\scriptstyle t=(T-d_{min}+1)..T,\ s=1..4} \tag{5}$$

$$\sum_{t'=t+1}^{t+dr-1} X_{s,t'} \leq dr(1 - X_{s,t-1} + X_{s,t}) \qquad {\scriptstyle t=2..(T-dr+1),\ s=1..4} \tag{6}$$

Constraints (2) are explained considering that I_s is the set of unavailability intervals for probe s. Constraints (3) state that the orbiter is a disjunctive resource; this prevents two communications from being planned at a time. Constraints (4) and (5) force each communication to last a minimal duration d_{min}. Constraints (6) force each probe to fulfil a reconfiguration time dr after a communication. Finally the objective function (1) is to maximize the total communication duration.

At present time, the Netlander mission is planned to operate with a single satellite. However the system may be able to work using other future satellites orbiting Mars (*e.g.* communication relay orbiters of the Italian Space Agency and the NASA). Hence we propose to generalize the previous model and to deal with a general assignment problem between a set of probes and a set of orbiters. The main modifications rely on adding a third index (related to the chosen orbiter) to decision variables, and to explicitly consider capacity constraints for the probes since a probe can communicate with one orbiter simultaneously (due to its unique antenna).

The Cplex solver has been used by way of C++ libraries of Ilog Concert. For one satellite the results are excellent since the optimal horizon is always obtained in less than 3 minutes for real-size instances over a horizon of 30 days. In the case of multi-satellites the optimal solution is also obtained in 2 up to 3 minutes for 2 satellites and a horizon of 15 days, 3 satellites and a horizon of 10 days, and 5 satellites and a horizon of 5 days. This is quite satisfactory.

Experiments planning

Experiments requested by scientists are to be realized by the probes. Thus, the second subproblem consists in programming these experiments over short horizons (typically a week). The corresponding operations plan needs to take into account ground and in-orbit communication capability as well as the availability at any time of the resources of the probes.

Each scientist is specifically concerned with a given instrument of a probe and has divergent views regarding the priorities of all experiments. Therefore the operations plan will have to be negotiated; it follows that the constraints and the objective function(s) for this second subproblem are not just as well defined as in the first subproblem.

Nevertheless we are sure that the number of experiments will be huge and the constraints numerous and heterogeneous (mutual exclusions, time windows, precedences, resource capacities). We then propose a decision-aid approach based on constraint propagation so as to reduce the search space of feasible solutions without explicitly taking account of any optimization criterion. The developed mechanisms implement specific resource constraint propagation mechanisms like *energetic reasoning* [Lopez, 96].

Pleiades

Problem description

At present time Earth observing satellites operated by the CNES belong to the Spot family. The Pleiades program is devoted to replace Spot satellites in the four next years. The objective is to have more *agile* satellites, that is, while a satellite moves on its orbit it can rotate in such a way the imaging instrument can make acquisitions in all directions (see figure 1).

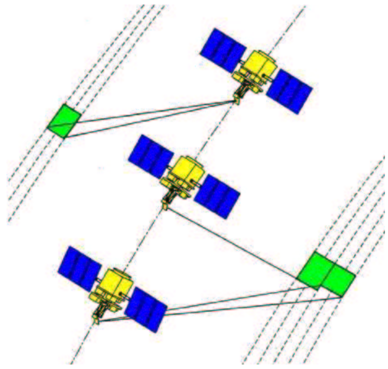

Figure 1: Example of images taken by a satellite

Observing satellites are operated from Earth in order to meet image requests of certain terrestrial zones. Considering a set of requests, the problem under study consists in selecting and scheduling the images taken by a satellite over a given horizon so as to maximize the customer satisfaction over this horizon. Due to the satellite agility and the huge number of requests (hundreds in 50 minutes, time corresponding to a half-revolution of the satellite over the enlightened side of the Earth) the complexity of the problem is very high.

A previous work has been done on this problem [Lemaître 02]. Several methods have been tested for the resolution: Tabu search is currently giving the best results; Linear Programming (Ilog Cplex) and Constraint Programming (Ilog Solver) have been also envisaged but without furnishing satisfying results with the retained model.

On another hand, other researches have been investigated on akin satellite management problems. For example [Paschos 01] proposes graph algorithms while [Gabrel 99] uses column generation. As in this latest work our goal is here to propose a column generation method to provide good upper bounds on the optimal solution.

Modeling

The main variables of the optimization problem are the following:

- a binary variable $sa(k, i)$ equal to 1 if image k is selected in position i, 0 otherwise.
- a continuous variable $ta(i)$ representing the start time of the image in position i in the built sequence.

For the description of the main constraints, one must before precise that every zone to be acquired is decomposed into rectangular areas named *strips* with constant and fixed width but of variable length. Moreover each selected strip j can be acquired in a direction or the opposite; the associated images are denoted by $2j$ and $2j + 1$ (see figure 2).

Though this feature is explicitly taken into account in the problem formulation, images $2j + 1$ will be rarely acquired. Indeed, knowing that slew rates for the satellites are so slow,

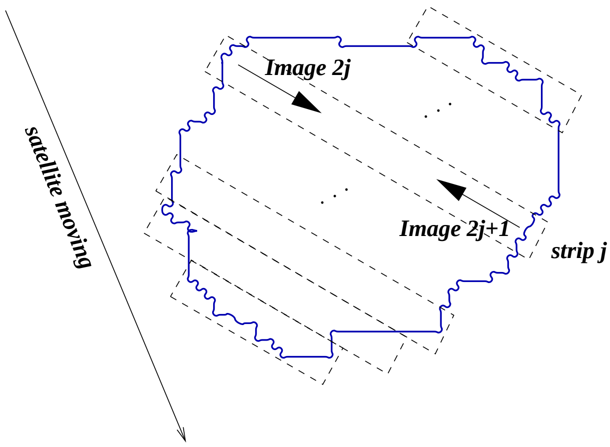

Figure 2: A request decomposed in strips

it is worth putting the instrument in a given orientation and then get the strip in the direction of satellite motion.

The problem can be formulated as follows:

$$\max \sum_r (G(r) * g(fr(r))) \quad (1)$$

$$\sum_i (sa(2j,i) + sa(2j+1,i)) \leq 1 \qquad \forall j \quad (2)$$

$$\sum_j (sa(2j,i) + sa(2j+1,i)) \leq 1 \qquad \forall i \quad (3)$$

$$\sum_j \underline{T}_1(j) * sa(2j,i) < ta(i) < \sum_j \overline{T}_1(j) * sa(2j,i) \qquad \forall i \quad (4)$$

$$\sum_j \underline{T}_2(j) * sa(2j+1,i) < ta(i) < \sum_j \overline{T}_2(j) * sa(2j+1,i) \qquad \forall i \quad (5)$$

$$ta(i+1) > ta(i) + Du(j) + \sum_l Dt(2j,2l) * sa(2l,i+1) + \sum_l Dt(2j,2l+1) * sa(2l+1,i+1) \qquad \forall i \quad (6)$$

$$\sum_i sa(2 * Str(r,2j+1),i) = \sum_i sa(2 * Str(r,2j+2),i) \qquad \forall r,j \mid St(r)=1 \quad (7)$$

$$\sum_i sa(2 * Str(r,2j+1)+1,i) = \sum_i sa(2 * Str(r,2j+2)+1,i) \; \forall r,j \mid St(r)=1 \quad (8)$$

Constraints (2) mean that each strip is acquired at most once, in either a direction or the opposite. Constraints (3) are so-called allocation constraints stating that each position in the resulting sequence is occupied by at most one image.

Due to satellite motion the image acquisitions are obviously subject to time windows. Constraints (4) and (5) represent these restrictions where a time window [*earliest-start-time*,*latest-start-time*] is given for every strip and for each direction of acquisition. We then make the assumption that image $2j$ can be acquired in $[\underline{T}_1, \overline{T}_1]$ while image $2j+1$ can be acquired in $[\underline{T}_2, \overline{T}_2]$.

Transition times are imposed between image acquisitions related to different satellite positions. Hence the beginning of two adjacent images in the sequence (j, l) must be separated by at least a distance equal to the duration of the first image, $Du(j)$, plus the transition time from j to l, $Dt(\text{image}(j), \text{image}(l))$. Constraints (6) illustrate the modeling of these transition times if image $2j$ is selected in position i.

Each stereoscopic request must be acquired twice in the same direction. This is illustrated by constraints (7) and (8) considering that with each request r is associated a Boolean $St(r)$ equal to 1 if request r is stereoscopic, 0 otherwise, and where $Str(r,.)$ is a pointer on the strips linked to request r.

Finally, searching for solutions is driven by criterion (1). The objective is to maximize an economic function taken from the customers' satisfaction. This function is computed on the basis of the reward obtained for the whole satisfaction of a request $G(r)$ weighted by a coefficient which can grow non-linearly (function g, see figure 3) with the acquired fraction of the request $fr(r)$.

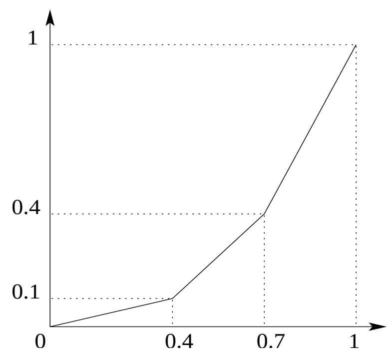

Figure 3: Example of a piece-wise linear weight function g

Column Generation

Introduction Problems involving satellite management have been already solved using column generation [Gabrel 99, Lebbar 00]. Moreover, the Pleiades problem has many common features with problems in the literature solved by column generation.

The idea of column generation is to work with a reasonably small subset of columns, *i.e.*, of variables [Lubbecke 02]. On the one hand, a *generation problem* derives optimal solutions to a subproblem which satisfies a restricted set of constraints. On the other hand, a linear program, called the *master problem*, weights the solutions previously generated and does an optimization on the relaxed problem. These two problems are solved iteratively exchanging information at each step until reaching the optimum of the master problem.

The Pleiades problem is close to the Vehicle Routing Problem with Time Windows (VRTPTW). Indeed, each image acquisition can be considered as a customer to visit. The

VRTPTW is successfully solved by column generation, for example in [Desrochers 92], more recently in [Chabrier 02]. In these works, the master problem is a covering problem and the generation problem solves a Shortest Path Problem with Time Windows (SPPTW). The main difference of the Pleiades problem with most routing problems concerns the selection of tasks to perform. Indeed, the number of requests is usually much higher than the number of images effectively acquired. In [Feillet 01] this particular *selective routing problem* is studied. A column generation is proposed, based on a new graph algorithm solving a "vehicle routing problem with profitable arcs" (VRPPA). In the VRPPA the number of resources is not fixed but minimized, whereas in Pleiades the observing satellite is the unique resource. However, column generation has been already proven to be an efficient way to solve single machine scheduling problems [van den Akker 00]; the problem is formulated like a time-indexed linear program in which the generation problem provides *pseudo-schedules*, *i.e.,* a task can be processed 0, 1, or several times. Then the master problem includes the fact that a task must be processed exactly once.

Problem Decomposition Inspired from the above works we propose to decompose the global problem into a master problem and a best-path problem in a graph, as follows. In order to get a linear master problem, we first simplify the criterion considering that g is the identity function in expression (1). The resulting criterion is linear and becomes:

$$\max \sum_r G(r) * fr(r) \quad = \max \sum_j \gamma_j * \sum_i (sa(2j,i) + sa(2j+1,i))$$

where γ_j is the reward obtained with acquisition of image $2j$ or $2j+1$.

The master problem consists of the strip acquisition constraints (2), allocation constraints (3), and stereoscopic constraints as well (7–8). The resulting master problem is a linear program which weights columns, each column corresponding to a *pseudo-sequence,* that is sequence of images that only satisfy temporal constraints (4–6), so as to maximize the sum of weighted rewards of columns. The number of pseudo-sequences is too large to explicitly consider all of them. Therefore we must devise an algorithm to generate the columns needed at each step of the resolution, that is, pseudo-sequences able to improve the criterion of the master problem.

Pseudo-sequences generation problem In this problem we search for sequences of images that satisfy temporal constraints (4–6). In these sequences each image must be acquired in its time window, and acquisitions of two successive images l and m in the sequence must be distant of a transition time $Dt(l,m)$.

Let $G = (N, A)$ be an oriented graph where N is the set of nodes representing the candidate images and A is the set of arcs representing the possible transitions between two images. With each node $l \in N$ is associated the time window $[\underline{T}_l, \overline{T}_l]$ and the positive duration $Du(l)$ of image l acquisition, and with each arc $(l,m) \in A$ is associated the positive transition duration between l and m, $Dt(l,m)$. Figure 4 gives an example of such a graph with five candidate images.

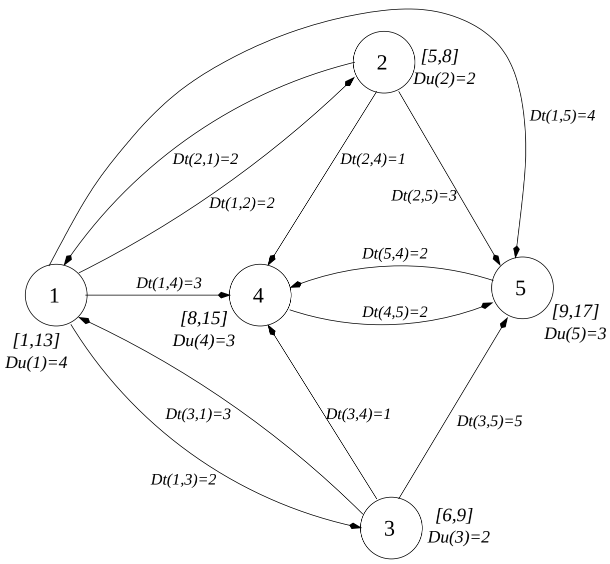

Figure 4: Graph representing five candidates images

Considering the linear criterion, we assume that with each image l is associated a reward g_l obtained for its acquisition. Hence, searching for pseudo-sequences able to improve the master problem consists in searching for longest elementary paths in graph G, with length $\mathcal{G}_p$ of a path p defined as a linear function of the rewards associated with the nodes in the path.

Let mark each path p in G with the label $\mathcal{L}_p = (l, \mathcal{D}_p, \mathcal{G}_p)$ where l is the last node of p and $\mathcal{D}_p$ the finish time of image l acquisition.

We introduce the following dominance criterion: Given two feasible paths p and p' in graph G (terminating on the same node) with associated labels $(l, \mathcal{D}_p, \mathcal{G}_p)$ and $(l, \mathcal{D}_{p'}, \mathcal{G}_{p'})$, we state that p dominates p' if and only if the three following conditions hold:

- $\mathcal{G}_p \geq \mathcal{G}_{p'}$ (*i.e.,* path p is longer than path p'),
- $\mathcal{D}_p \leq \mathcal{D}_{p'}$ (*i.e.,* acquisition of l ends sooner in path p than in path p'), and
- $p \subseteq p'$ (*i.e.,* each image acquired in path p is acquired in path p' as well).

We add two nodes to graph G: a source s and a sink q, both associated with a null reward. The principle of the algorithm for the pseudo-sequences generation is to construct differents paths starting with node s and to maintain a *set of dominant paths* according to the above definition of dominance. It is described in Algorithm 1.

Results We obtained first results in a particular context where a maximum computation duration of 300 seconds is allowed. In this short computation duration a column generation method of resolution, like any exact method, cannot be efficient on this problem. Hence we implemented a

Algorithm 1 Pseudo-sequence generation

```
S ← {(s, 0, 0)}
NewDominantPaths ← ∅
(l, D, G) ← choose_best(S)
repeat
   repeat
      for each node j successor of l do
         extend path (l, D, G) to new path (j, D′, G′)
         if (j, D′, G′) is dominant then
            insert (j, D′, G′) in set S
            update S
            insert (j, D′, G′) in set NewDominantPaths
         end if
      end for
      (l, D, G) ← choose_best(NewDominantPaths)
      NewDominantPaths = ∅
   until (l = q)
   (l, D, G) ← choose_best(S)
until (l = q)
```

heuristic version of the algorithm proposed for the pseudo-sequences generation, without considering the master problem optimisation steps. This algorithm integrates new constraints, in particular the stereoscopic ones and the non-linear maximization criterion. The length of a path is now computed as the sum of rewards associated with the non-linearly-weighted acquired fraction of each request, and so the dominance criterion cannot guarantee that "good" paths would not be eliminated during the iterative process.

For small instances, the obtained solutions are close to the reference results given in [Lemaître 02]. On the largest instances, our results are worse, due to the growing number of eliminated "good" paths by the dominance criterion. This part is currently improved.

Further Works

In this paper we are concerned with two complex optimization problems issued from space applications. In the first one, the Netlander mission, we have shown that the communications planning can be rather easily solved to optimality using linear programming. In the second one, the Pleiades project, we have to face with a high combinatorial problem since the planning of images at hand can be reduced to an NP-hard scheduling problem.

The work on Pleiades is still on progress. Our main objective is now to get solutions of the problem with linear criterion with the proposed column generation algorithm. Indeed column generation has been proven to be an efficient exact method for large-scale combinatorial problems close to the Pleiades problem. Therefore column generation seems a promising way to provide a good upper bound on the problem criterion by solving to optimality a linear relaxation of the global problem.

We will also intend to integrate the non-linear computation of paths length in the pseudo-sequences generation algorithm. This would allow the consideration of the non-linear criterion of the problem in the column generation method, and thus, to provide good approximate solutions to this problem.

Finally the implementation of the column generation algorithm will be used for the multi-orbiters Netlander mission. Tests are to be carried out with data including 5 orbiters and a planning horizon up to 30 days.

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