Using Simulation for Execution Monitoring and On-line Rescheduling with Uncertain Durations

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Abstract

The problem we tackle is on-line rescheduling with temporal uncertainty, activity durations are uncertain and activity end times must be observed during execution. In this paper we will assume we have a representation of the uncertainty of each activity duration in the form of probability distributions which are used in the simulation of schedule execution. We use the simulations to monitor the execution of the schedule and in particular to estimate the quality of the schedule and the end times of the activities. Given an initial schedule, the schedule starts execution and we must decide when to reschedule. We propose and explore a non-monotonic technique where each time we reschedule we can completely change the existing schedule except for those activities that have already started (or finished) execution. This paper explicitly addresses the basis on which the decision to reschedule is made by investigating three simple measures of the data provided by simulation.

Introduction

Nowadays planning and scheduling must take into account incomplete information and/or potential changes in the environment, i.e. uncertainties (Smith, Frank, & Jónsson 2000). The central issue is to design robust planning and scheduling techniques, aimed at guaranteeing the feasibility and the quality of the executed schedule. Several ways to get a more robust schedule have already been investigated (Herroelen & Leus 2002; Davenport, Gefflot, & Beck 2001; Davenport & Beck 2000; Wu, Byeon, & Storer 1999; Le Pape 1995; Smith 1994). One way among others is to keep one and only one fixed schedule to execute, but reschedule when it appears the quality of the currently executing schedule degrades. This approach is relevant as far as rescheduling is fast enough w.r.t. the scheduling execution, i.e. when the dynamics of the system are low. Relevant questions are then: how and when the quality should be assessed? How and when we should reschedule? How do we assess the robustness of the approach? The paper intends to partially answer these questions.

The problem we tackle is on-line rescheduling with temporal uncertainty, activity durations are uncertain and activity end times must be observed during execution. In this paper we will assume we have a representation of the uncertainty of each activity duration in the form of probability distributions which are used in the simulation of schedule execution. We use the simulations to monitor the execution of the schedule and in particular to estimate the quality of the schedule (i.e., the makespan) and the end times of the activities. In the technique that we propose and explore, those data are used to decide when to reschedule; when such a decision is taken, the initial schedule is forsaken and a new one is computed, except for those activities that have already started (or finished) execution. The global process is hence clearly a non-monotonic on-line scheduling strategy which is used in a reactive way. This paper addresses the basis on which the decision to reschedule is made by investigating three simple measures of the data provided by simulation.

Definitions

In this paper we focus on the problem of execution monitoring and the decision to reschedule for a Job-Shop Scheduling Problem (JSSP) with uncertain durations. In this section, we define the JSSP, present our assumptions about the evolution of the uncertainty of activities during execution, and give a number of definitions used throughout the paper.

A JSSP is composed of jobs. A job is a set of activities that have to be performed in a given order i.e. there is a precedence constraint between each pair of activities in this set. Each activity is assigned to a single resource. Each resource can process one activity at a time, resources are modeled as unary resources (resource capacity of unit capacity). A solution to a JSSP is the assignment of a start time to each activity in order to satisfy the temporal and resource constraints and to optimize one or several criteria. Our objective, which is often used, is to minimize the makespan of the JSSP i.e. to minimize the time between the start time of the first activity executed and the end time of the last activity executed. A JSSP is a generic problem formulation suitable for many application problems, not only resources in a shop, namely when one has specific sequences of activities requiring resources. Hence a JSSP may be a sub-problem of a more general planning problem.

Each uncertain activity duration is modeled by a variable that follows a probability distribution and is associated with a domain. Any probability law can be used (normal, uniform, etc.) however as the duration of an activity must be strictly positive, the laws are truncated. At execution, we know the effective duration of an activity only when the activity ends and we learn this piece of information from the environment. The only decision is when to start the execution of each activity. As a consequence, we know at each moment, during the execution which are the activities that have been executed, the ones that are currently executed, and the ones that have not been started yet. When an activity is still executing and its minimum possible duration has been exceeded, our information on the uncertainty can be updated since the set of possible durations is now reduced. Therefore the probability distribution is further truncated and renormalized. We update data at specific points in time, i.e. the moments when we want to decide if we reschedule or not, and hence we need to know the precise current state.

If the initial probability law of an activity is described by a distribution function $p_0(d)$, and if the activity has been executed since $d0$ units of time, the current distribution is defined by the probability distribution $p_{d0}(d)$ as follows:

$$\forall d \in [0, d0], p_{d0}(d) = 0 \tag{1}$$

$$\forall d > d0, p_{d0}(d) = \frac{p_0(d)}{\int_{d0}^{+\infty} p_0(t)dt} \tag{2}$$

The following three definitions are used in this paper:

- The **effective** duration of an activity is the duration that is observed during execution. The effective schedule is the sequence of activities obtained after execution: all the activity durations have been observed. An **execution scenario** is the set of all effective durations.
- An **indicative** schedule is a solution that is generated by using fixed activity durations. In an indicative schedule the activities on each resource are totally ordered. The first indicative schedule is calculated off-line by using the mean activity durations. During execution a new indicative schedule is constructed if we decide to reschedule; in that case this new indicative schedule is calculated by using both the effective durations of the activities already executed and the mean durations of the other activities.

Non-monotonic Approach

Execution Model Description

The initial (off-line) schedule starts execution and we must decide when to reschedule. The main idea behind the criteria investigated here is that we use simulation to determine when to reschedule. We start execution using the indicative schedule and a simple execution policy: activities are started as soon as possible following the precedence constraints in the original problem, the activity sequences defined by the indicative schedule, and the effective activity durations that are observed. During execution of the schedule, we choose to evaluate the need for rescheduling whenever an activity ends. The first step consists then in updating the truncated probability distributions representing our view of the uncertain variables, as discussed in the last section. The second step is to calculate a chosen rescheduling criterion that depends on estimated values of some variables in the problem. If the estimated value varies too greatly from the indicative value, we reschedule.

Our estimations of the values for the variables we are interested in are generated through the use of simulation. According to the currently executing schedule and the current probability distributions, we randomly generate 1,000 execution scenarios whenever an activity ends. Each scenario allows us to calculate the simulated values for the variables we are monitoring and over the 1,000 simulations we calculate their means and standard deviations. The estimated value of the variable is its mean value over the simulations. If we decide to reschedule, the current schedule is ignored and a new schedule is generated except for the activities that have already started or finished executing.

Three Variations

The first criterion consists in monitoring the makespan and is defined as follows: we reschedule when the estimated makespan M_{est} is greater than the rescheduling threshold, $M_{est} > \frac{M_{ind}}{\sigma}$ where M_{ind} is the indicative makespan and σ is the sensitivity factor. This criterion will not result in rescheduling if activity durations are shorter than expected. In other words, it does not allow “opportunistic” rescheduling that would take advantage of unexpectedly short execution times.

A second variation, based on the first, is opportunistic because it reschedules based on the absolute difference between the estimated and indicative makespans. Rescheduling is done when the absolute difference between the estimated makespan M_{est} and the indicative makespan M_{ind} is greater than the rescheduling threshold. In this case, our rescheduling threshold is the mean of all activity durations of the deterministic problem, D, divided by σ. More formally, we reschedule when: $|M_{est} - M_{ind}| > \frac{D}{\sigma}$.

The two versions based on makespan monitoring are *a priori* rather crude: we mainly take into account the observed durations of the activities of the critical path. If these activities do not slip too much then, as a consequence, the makespan will not slip too much either. It is possible however, that the estimated makespan remains approximately stable while the executions of the activities that do not belong to the critical path are such that it is possible to find a much better solution by rescheduling. We thus propose a third variation of the approach that takes into account each activity duration. First we consider at each time the set A of the n activities that had not finished execution the last time we rescheduled. $\overline{|D_{diff}|}$ is then defined as the mean of the absolute differences between the estimated and indicative end times denoted end_{est} and end_{ind} for such activities: $\overline{|D_{diff}|} = \frac{\sum_A |end_{est} - end_{ind}|}{n}$. We reschedule each time $\overline{|D_{diff}|}$ is greater than our previous rescheduling threshold, i.e. when $\overline{|D_{diff}|} > \frac{D}{\sigma}$.

Experimental Study

The instances of the JSSP with imprecise durations are generated from classical JSSP instances. Each activity is associated with a normal probability distribution with mean, p, corresponding to the duration in the classical instance, and with standard deviation $\alpha \times p$ with $\alpha > 0$. α characterizes the degree of uncertainty of the problem. The higher α the higher the standard deviation of each activity so the more uncertainty in the system. α is constant and equals 0.3 for each experiment done in this study.

During schedule execution, whenever we are informed by the environment that an activity has ended, we update all our data structures, and simulate the continued execution of the schedule. The updating frequency is sufficiently low to permit us to run 1,000 simulations each time. When the end of an activity is observed, there might be other concurrent activities still being executed for which distributions are updated. Scheduling and rescheduling are done using the constraint programming approach with a standard chronological backtracking algorithm and a time limit of one second. The new schedule is the best solution found within the time limit. On the tested problem instances, the optimal schedule was often found. All scheduling is done with ILOG Scheduler 5.3 (ILOG 2002). Unless otherwise noted, the results come from running 100 different execution scenarios per value of the sensitivity factor. In this experimental study we only experiment with one problem, future work will explore a set of problems.

Results

We use a measure of the computational effort and schedule quality to evaluate our rescheduling criteria. The former is measured by the number of times that we reschedule, while for the latter we use the estimated makespan. The problem we tackle here is called la11 (Lawrence 1984). It is a JSSP with 20 jobs and 5 resources. The optimal makespan with the mean durations is 1,222.

Makespan Monitoring

In this subsection we display the results for the first variation when the makespan is monitored.

Figure 1 and 2 represent the mean and the standard deviation of the number of reschedulings as the sensitivity factor σ changes.

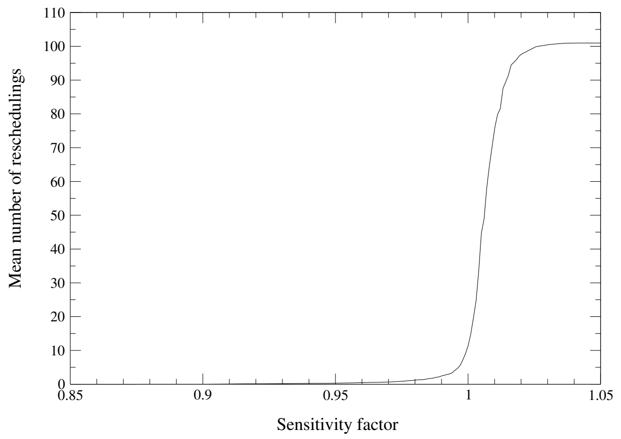

Figure 1: Mean number of reschedulings with makespan monitoring

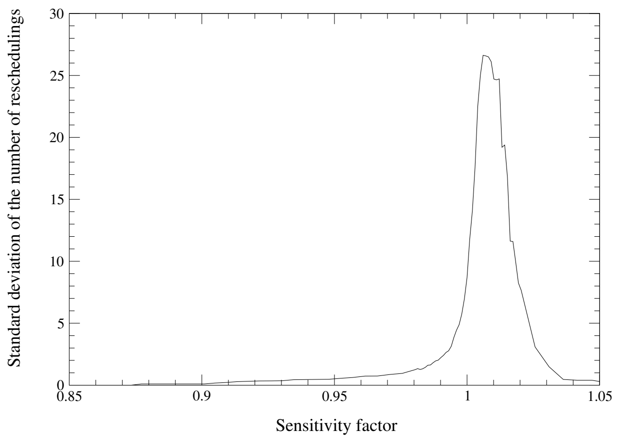

Figure 2: Standard deviation of the number of reschedulings with makespan monitoring

Figure 3 and 4 represent the mean and standard deviation of the estimated makespan versus the sensitivity factor σ.

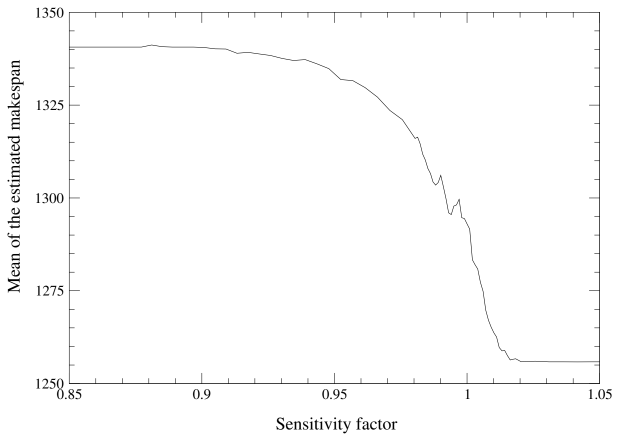

Figure 3: Mean of the estimated makespan with makespan monitoring

Absolute Makespan Monitoring

Turning to our second criterion, we expect to find better results if we also reschedule when activities end earlier than estimated.

Figure 5 and 6 represent the mean and the standard deviation of the number of reschedulings versus the sensitivity factor σ.

Figure 7 and 8 represent the estimated makespan versus the sensitivity factor σ. The mean and standard deviation of the estimated makespan are plotted.

Monitoring of Activity End Times

The first two criteria are myopic because they only focus on the makespan. Makespan is an aggregate view of the "state" of the schedule since it depends only on the activities on

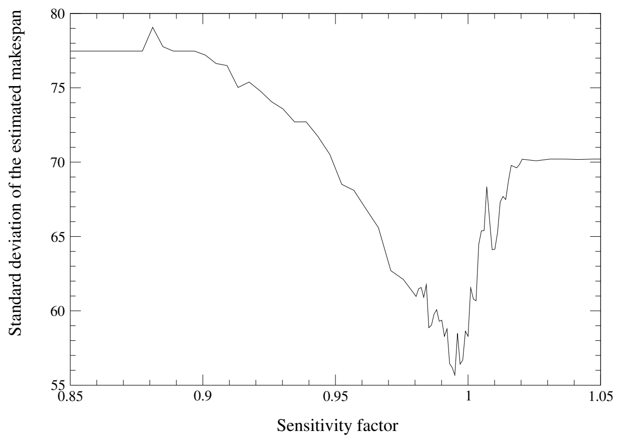

Figure 4: Standard deviation of the estimated makespan with makespan monitoring

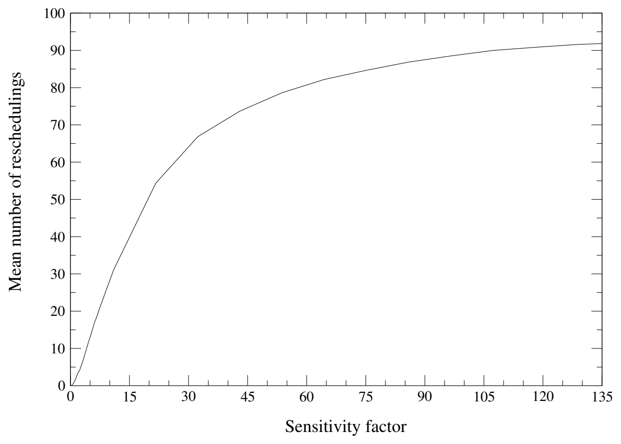

Figure 5: Mean number of reschedulings with absolute makespan monitoring

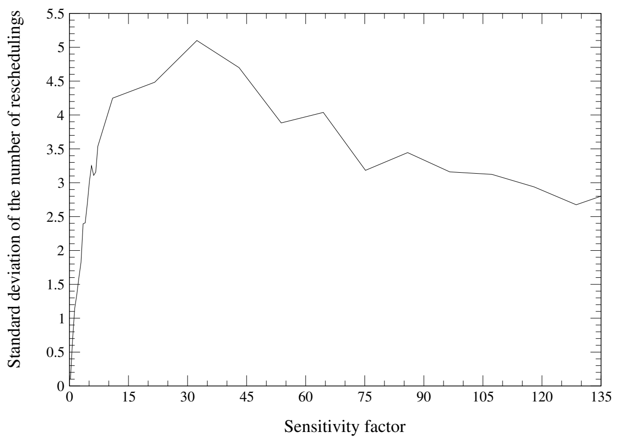

Figure 6: Standard deviation of the number of reschedulings with absolute makespan monitoring

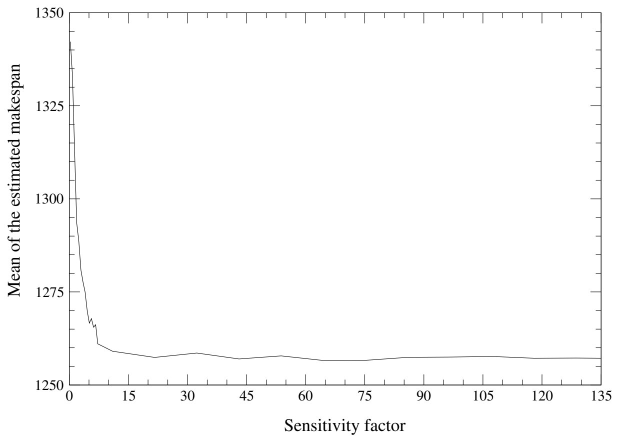

Figure 7: Mean of the estimated makespan with absolute makespan monitoring

the critical path. In addition, when the standard deviation of the estimated makespan is large, the mean of the estimated makespan does not provide accurate information. We thus investigate a third rescheduling criterion that takes into account all the activity durations individually.

For the same execution scenarios as those used for Figure 5, 6, 7, and 8, the shapes of the curves representing the mean and the standard deviation of the number of reschedulings versus the sensitivity factor σ are very similar to those on Figure 5 and 6; the shapes of the curves representing the mean and the standard deviation of the estimated makespan versus σ are very similar to those on Figure 7 and 8.

These results show that this rescheduling criterion performs better than the absolute makespan criterion because we are now opportunistic w.r.t. each activity execution (not only w.r.t. the activities belonging to the critical path). We thus observe that the effective (final) makespan is always less than the first estimated makespan $M_{est} = 1,349.87$ and the higher sensitivity factor, the lower the final makespan with more reschedulings.

Global comparison

Figure 9 shows the mean estimated makespan obtained for each of the three variations versus the mean number of reschedulings. Whatever the rescheduling criterion, monitoring permits to improve schedule quality. These three curves confirm that the criterion based on the activity end time monitoring is almost always the best in terms of quality: for a given computational effort (here the mean number of reschedulings greater than 20), this rescheduling technique provides the smallest makespan.

Future Work: Monotonic Approach

We would also like to investigate another approach which only performs monotonic decisions, that is, unlike the first approach, when a given decision has been taken to order activities, it cannot be changed later on; remember that in the approach investigated in this paper, the indicative schedule

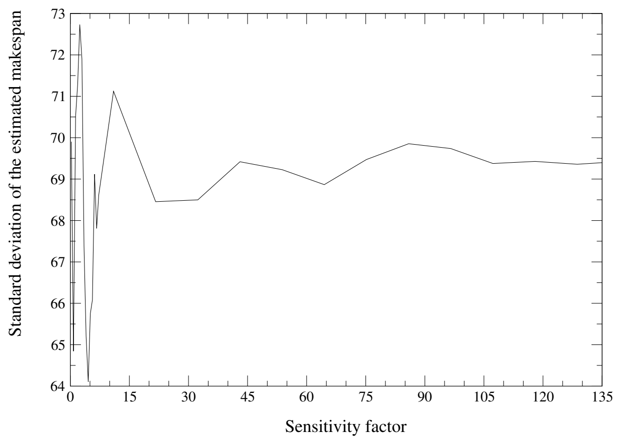

Figure 8: Standard deviation of the estimated makespan with absolute makespan monitoring

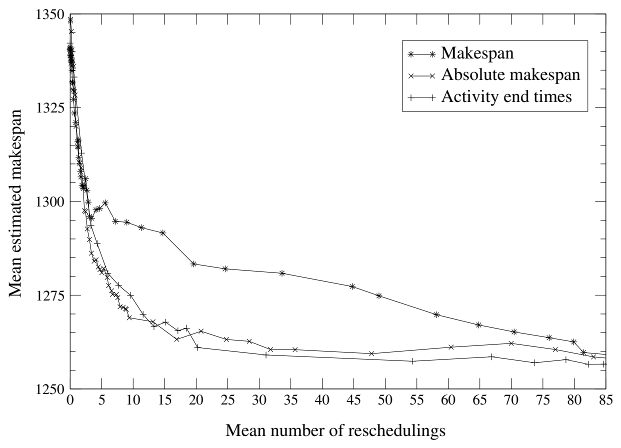

Figure 9: Mean estimated makespan for the three criteria

could be recomputed from scratch. The problem with such a monotonic approach is that we must be very careful when taking a decision. On the one hand we have to wait until uncertainty around the decision is low enough so that the decision is well informed. On the other hand we cannot wait too long because the schedule is executing and we do not just want to have a reactive and myopic decision process. In this approach, we have decided to schedule the overall problem piece by piece during the execution. Determining when to schedule a new set of activities and how to select the set to be scheduled will be done using information from simulation. This approach is particularly suited for applications where the dynamics are high.

Conclusion

This paper presents a non-monotonic approach to deciding when to reschedule based on the use of simulation. We simulate on-line, at specific points in time, the execution of the remainder of the indicative schedule. Three rescheduling criteria were explored. According to the results, rescheduling permits to improve schedule quality. The computational effort necessary to reach a given schedule quality depends on the rescheduling criterion. This work in progress seems to be promising since there are a lot of new directions we can follow to better understand and tune our techniques in order to adapt to different applications.

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