Deployment and Maintenance of a Constellation of Satellites: a Benchmark

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Abstract

This paper presents the problem of the management of the deployment and of the maintenance of a constellation of satellites, as it has been set by the French space agency (CNES). After an informal presentation, a more formal description, based on the Markov Decision Process (MDP) framework, is provided. Several approaches for dealing with such a problem are then emphasized. We think that such a real life problem might make up a challenging benchmark for any approach in the domain of automatic decision and planning under uncertainty.

Introduction

In this paper, we want to present as precisely as possible the problem of the management of the deployment and of the maintenance of a constellation of satellites, independently of the type of satellite and of the type of mission (telecommunication, navigation, or observation), as it has been set by the French space agency (CNES).

We do that because we think that such a real life problem might make up a challenging benchmark for any approach in the domain of automatic decision and planning under uncertainty.

In the next section, we provide the reader with a global informal view of the problem. In the following section, we provide her/him with a more formal description. Any formal description requires a framework. In that case, we use as a basis the *Markov Decision Process* (*MDP*) framework (Puterman 1994). Decision instants and temporal horizon, states, constraints on states, initial and goal states, decisions, constraints on decisions, effects of actions, probabilities of transitions, local and global costs are successively defined. In the last section, several approaches are evoked without any claim to be exhaustive.

Global view of the problem

Whatever its mission is (telecommunication, navigation, or observation), a constellation of satellites is made up of a specified number of spatially distributed spacecraft, which together allow the mission to be filled. All the satellites or at least a subset of them must be operational to satisfy the mission objectives. If too few satellites are operational, the mission objectives will be only partially met, and eventually not at all.

But, in general, all the satellites cannot be launched at the same time by the same launcher. Several launches, using eventually various launcher types and various launch sites, are necessary. These launches must be organized over time.

Moreover, failures may occur at any stage of the deployment, of the maintenance, and of the operational life of the constellation. So, the management of its deployment and of its maintenance must be able to anticipate these possible failures, as well as to react to them when they occur.

Globally speaking, managing the deployment and the maintenance of a constellation consists in organizing the launches and the orbital transfers in order to deploy it as soon as possible and to maintain it as best as possible in its operational state.

More precisely, the constellations we consider are organized along several orbital planes (see Figure 1). A specified number of operational satellites is necessary on each orbital plane. On each orbital plane, satellites may be either on an operational orbit, or on a spare orbit. Satellites that are on a spare orbit are drifting in a month from an orbital plane to the following one. Launchers are able to put a specified number of satellites on one of the orbital planes: all the launched satellites on the same orbital plane. These satellites can be either immediately transferred from the spare orbit to the operational one on this orbital plane, or left on the spare orbit to drift from orbital plane to orbital plane. In the later case, when their orbital plane coincides with an operational orbital plane, that is once per month, they may be transferred from the spare orbit to the operational one on this orbital plane (see Figure 2).

Launches are not possible at any time. We consider that no more than one launch is possible each month and that a minimum time must pass between two launches of the same type. This minimum time increases in case of failure of the first launch, in order to let enough time for inquiry. Moreover, management of the launch sites imposes that launches can be neither decided nor cancelled at the last minute: a launch must be either decided or cancelled a specified time in advance, except in case of failure of a launch, which may impose to postpone launches of the same type, in order to meet the minimum time between launches after failure. Constraints on the production of launchers and satellites

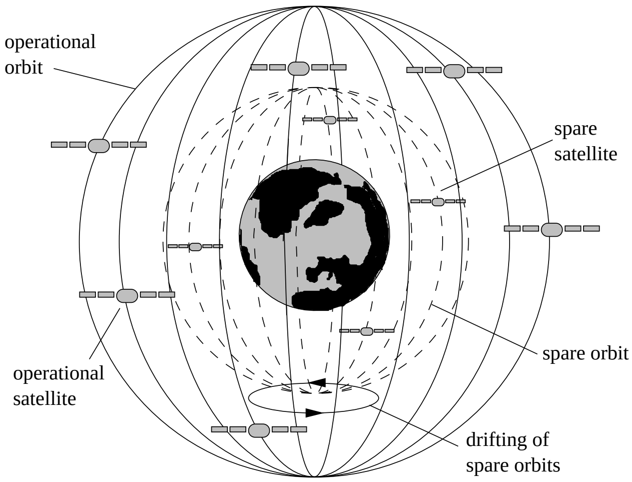

Figure 1: View of the goal constellation.

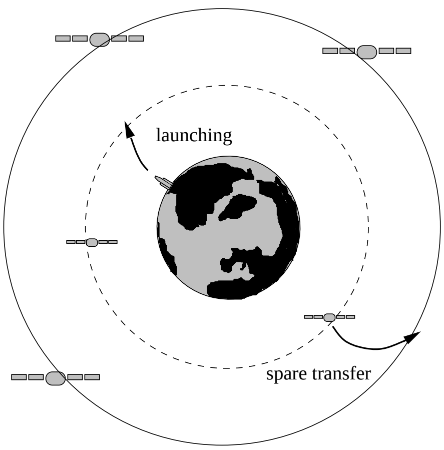

Figure 2: On an orbital plane, launch of a satellite and transfer of a spare satellite from the spare orbit to the operational one.

are not considered here.

Two types of cost must be considered: firstly, the cost of the production of launchers and satellites and of the launches; secondly the cost which may result from a partial or complete unavailability of the constellation.

Failures may occur at any stage and at any time: launcher failure, spare satellite running failure, spare satellite orbital transfer failure, operational satellite running failure, failure of either a spare or an operational satellite.

The global objective of the management is finally to minimize over a given temporal horizon (either finite or infinite) the sum of the production and of the unavailability costs.

An MDP formulation

In this section, we want to provide the reader with a mathematically well-founded definition of the problem. Because the problem to solve is clearly what is referred to as a *sequential decision problem under uncertainty*, we use to model it the MDP framework (Puterman 1994), which is very convenient for modelling such problems.

Note immediately that we do not need the extension of the MDP framework to partial observability, referred to as *Partially Observable MDP* (*POMDP*) (Kaelbling, Littman, & Cassandra 1998), because we assume that the real state of the constellation is completely and precisely known at each decision time.

What we precisely use is a structured variant of the MDP framework, which is inspired but different from the *Factored MDP* framework (Boutilier, Dean, & Hanks 1999; Boutilier, Dearden, & Goldszmidt 2000): we use *state*, *decision*, and *random variables* for describing states, decisions and state transitions and we use *constraints* (disjunctions of equations, inequations, and disequations) for defining relations between these variables.

Note that this modelling choice does not prejudge the use of a specific solving approach. The best solving approach remains to be chosen among many candidates: Artificial Intelligence (AI) planning algorithms (Fikes & Nilsson 1971; Blum & Furst 1997); probabilistic planning (Kushmerick, Hanks, & Weld 1995); MDP algorithms like dynamic programming, value iteration, policy iteration (Puterman 1994); reinforcement learning (Sutton & Barto 1998); neuro-dynamic programming (Bertsekas & Tsitsiklis 1996); stochastic optimization (Fu 2001); case-based reasoning (Jona & Kolodner 1992); expert systems (Hayes-Roth 1992) ... See the last section for a short discussion about these various alternatives.

An MDP is usually defined by a quintuple $\langle I, S, A, T, R \rangle$, where I is a finite or infinite set of *instants* at which decisions must be made, S is a finite set of possible *states* of the system, A is a finite set of possible *decisions* of action on this system, T is a *state transition function* which associates, with any pair of states $s, s' \in S$ and each action $a \in A$, the probability to be in state s' after being in state s and applying action a, and R is a *transition reward function* which associates, with any pair of states $s, s' \in S$ and each action $a \in A$, the immediate gain or cost for being in state s' after being in state s and applying action a.

We describe these elements in the five sections that immediately follow the next one, which is dedicated to problem data.

Problem data

We introduce the following notations for the problem data about goal constellation, launching capacities, delays between launches, decision horizon, failure probabilities, and costs.

Goal constellation

- NOP is the number of orbital planes in the constellation;
- NOS is the number of operational satellites per orbital plane in the goal constellation.

Launching capacities

- NTL is the number of available types of launcher;
- for each type of launcher tl, $0 \leq tl \leq NTL$, $NLS[tl]$ is the number of satellites that can be launched by a launcher of type tl; $tl = 0$ means no launcher; $NLS[0] = 0$.

Delays between launches

- for each type of launcher tl, $0 \leq tl \leq NTL$, $MTL[tl]$ is the minimum time, in months, between two successive launches of type tl; $MTL[0] = 1$;
- for each type of launcher tl, $0 \leq tl \leq NTL$, $MTLF[tl]$ is the minimum time, in months, between two successive launches of type tl in case of failure of the first one; $MTLF[0] = 1$; moreover, $\forall tl$, $0 \leq tl \leq NTL$, $MTL[tl] \leq MTLF[tl]$.

Decision horizon

- DH is the minimum time, in months, a launch may be decided or cancelled in advance, that is the time the launching plan is fixed, except in case of failure of a launch, which may impose to postpone planned launches of the same type.

Failure probabilities

- for each type of launcher tl, $0 \leq tl \leq NTL$, $PFL[tl]$ is the probability of failure of a launcher of type tl; $PFL[0] = 1$;
- $PFRSS$ is the probability of failure when running a satellite on a spare orbit;
- $PFROS$ is the probability of failure when transferring a satellite from a spare orbit to an operational one and then running it on this operational orbit;
- for each satellite age a, $a \geq 0$, expressed in months, $PFSS[a]$ is the probability of failure over a month of a satellite of age a on an spare orbit;
- for each satellite age a, $a \geq 0$, expressed in months, $PFOS[a]$ is the probability of failure over a month of a satellite of age a on an operational orbit.

Costs

- for each type of launcher tl, $0 \leq tl \leq NTL$, $CL[tl]$ is the cost of a launcher of type tl; $CL[0] = 0$;
- CS is the cost of a satellite;
- $CU[]$ is the function that returns the cost per month of the complete or partial unavailability of the constellation, that is of the number of missing operational satellites on each orbital plane;
- γ is the discounting multiplicative factor to apply each month to future costs with regard to current ones.

Decision instants and temporal horizon

Because the drift of a spare satellite from an orbital plane to the following one takes one month and because one launch (and no more than one) is possible each month, we associate a decision instant with each month and assume that executing this decision and observing its actual effect on the constellation state take no more than one month.

The length of the temporal horizon to consider is either *finite* (limited to a specified time EH, in months), or *infinite*.

States, constraints on states

For each instant (each month of the temporal horizon), the state of the constellation obviously involves the number of satellites on each orbital plane, on the spare and on the operational orbit.

A first difficulty occurs with the age of the satellites. Introducing them to the state increases dramatically the dimension of the state space, but may be necessary for some more precise reasonings, especially in the maintenance phase, when satellite ages and then failure probabilities may be very different from each other.

A second difficulty occurs, due to the fact that a minimum time must pass between two launches of the same type and that a launch must be either decided or cancelled at least DH months in advance. The easiest way of taking these requirements into account in an MDP formulation, that is of maintaining the markovian nature of the process, is to introduce the current launching plan over DH months, as well as the current constraints on its possible extensions, to the state.

Finally, for each instant i, the state we consider involves six types of variable:

- for each orbital plane op, $1 \leq op \leq NOP$, the number $nss[i, op]$, $nss[i, op] \geq 0$, of spare satellites on this orbital plane;
- for each orbital plane op, $1 \leq op \leq NOP$, the number $nos[i, op]$, $nos[i, op] \geq 0$, of operational satellites on this orbital plane;
- for each orbital plane op, $1 \leq op \leq NOP$, the sequence $ass[i, op]$, ordered according to a decreasing order, of the ages of the spare satellites on this orbital plane;
- for each orbital plane op, $1 \leq op \leq NOP$, the sequence $aos[i, op]$, ordered according to a decreasing order, of the ages of the operational satellites on this orbital plane;

- for each instant i', $0 \leq i' \leq DH$, of the current decision horizon, expressed in months in relation to i (i' points out the instant $i + i'$), the type of launch $ptl[i, i']$, $0 \leq ptl[i, i'] \leq NTL$, that is planned at this instant; $ptl[i, i'] = 0$ means that no launch is planned at this instant;
- for each type of launcher tl, $0 \leq tl \leq NTL$, the minimum time $mtl[i, tl]$, $DH + 1 \leq mtl[i, tl] \leq DH + MTL[tl]$, in months, before a launch of this type can be planned, taking into account previous and future launches.

  Note that constraints link the last two types of variable:

$$\forall i', i'', 0 \leq i', i'' \leq DH, i' < i'', \\ (ptl[i, i'] = ptl[i, i'']) \Rightarrow (MTL[ptl[i, i']] \leq i'' - i') \tag{1}$$

$$\forall i', 0 \leq i' \leq DH, \\ MTL[ptl[i, i']] \leq mtl[i, ptl[i, i']] - i' \tag{2}$$

They express that the minimum time between launches of the same type must be met between every pair of planned launches of the same type, and between every planned launch and every launch of the same type that might be planned in the future. Other constraints should express that the cardinality of $ass[i, op]$ and $aos[i, op]$ are respectively equal to $nss[i, op]$ and $nos[i, op]$.

Decisions and constraints on decisions

Three types of decision must be made at each instant:

- the first one is related to the target orbital plane for the current launch, at instant i, if such a launch has been planned;
- the second one is related to the current orbital transfer, at instant i, of spare satellites from spare orbits to operational ones;
- the third one is related to the extension of the current launching plan: what type of launch, eventually no launch, to plan for instant $i + DH + 1$, which comes just after the last instant $i+DH$ of the current launching plan? (see Figure 3)

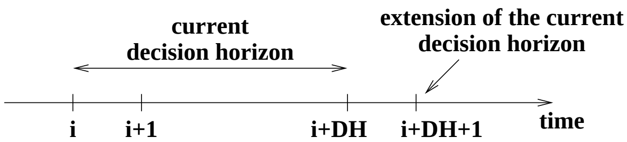

Figure 3: Decision horizon and its extension.

We assume that, at each instant i, these three types of decision are made in sequence in this order and that the effects of a decision can be observed before making the following one. This justifies that three states be associated with each instant. We note $\langle i, d \rangle, d \in \{1, 2, 3\}$ the state of the constellation at instant i just before making the decisions of type d (see Figure 4).

For each instant i and each type of decision d, the decision we consider involves the following types of variable:

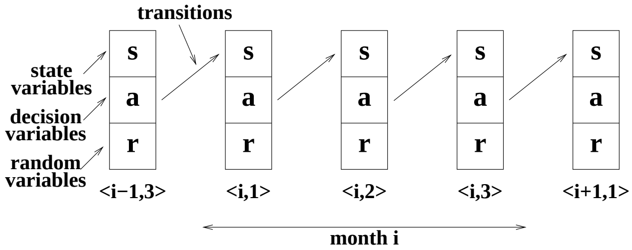

Figure 4: Each month, a three-step decision.

- for the first decision type, the orbital plane $lop[i]$, $0 \leq lop[i] \leq NOP$, to choose for the current launch, at instant i, if such a launch has been planned; $lop[i] = 0$ applies when no launch has been planned;
- for the second decision type, for each orbital plane op, $1 \leq op \leq NOP$, the number $nts[i, op]$, $nts[i, op] \geq 0$, of spare satellites to transfer, at instant i, from the spare orbit to the operational one, on this orbital plane, and, more precisely, the sequence $ats[i, op]$, ordered according to a decreasing order, of the ages of the spare satellites to transfer;
- for the third decision type, the type of launcher $etl[i]$, $0 \leq etl[i] \leq NTL$, to choose for extending the current launching plan to instant $i + DH + 1$.

Note that, whereas the effects of the first two types of decision are immediate, the effect of the last one is delayed.

Various constraints link state and decision variables and express that, for each type of decision, not all the decisions are possible in a given state:

$$(ptl[\langle i, 1 \rangle, 0] = 0) \Leftrightarrow (lop[i] = 0) \tag{3}$$

$$\forall op, 1 \leq op \leq NOP, \\ nts[i, op] \leq nss[\langle i, 2 \rangle, op] \tag{4}$$

$$mtl[\langle i, 3 \rangle, etl[i]] = DH + 1 \tag{5}$$

The first one (equation 3) associates a null orbital plane with a null type of launcher (no launch). The second one (equation 4) expresses that, on each orbital plane, it is not possible to transfer more satellites than the number of satellites that are available at this time on the spare orbit. The third one (equation 5) expresses that the minimum time between launches of the same type must be met between the launch that is chosen for extending the current launching plan and the previous launch of the same type. Other constraints should express that $ats[i, op]$ is a subsequence of $ass[\langle i, 2 \rangle, op]$ and that its cardinality is equal to $nts[i, op]$.

Effects of actions and probabilities of transitions

To describe the non deterministic transitions that result from a decision of action, we use random variables. Actual values of these variables, together with the values of the state and decision variables at instant i, completely determine the values of the state variables at the next instant $i+1$. But, whereas values of the decision variables are under the control of the management system, actual values of the random variables are out of its control. The only knowledge about their possible values is a probability distribution which is assumed to exist for each of them.

The random variables we use are:

- for the first transition type after the first decision step:
  - the failure $fl[i]$, $fl[i] \in \{0,1\}$, of the current launch, at instant i, if such a launch has been planned, with $fl[i]=1$ in case of failure;
  - the number $nfrss[i]$, $0 \leq nfrss[i] \leq NLS[tl[i,0]]$, of satellites that are launched at instant i and fail when running on the spare orbit; $nfrss[i] = NLS[tl[i,0]]$ when $fl[i]=1$;
- for the second transition type after the second decision step:
  - for each orbital plane op, $1 \leq op \leq NOP$, the number $nfros[i,op]$, $0 \leq nfros[i,op] \leq nts[i,op]$, of spare satellites that are transferred from the spare orbit to the operational one at instant i and for which either transfer or running on the operational orbit fails; more precisely, for each element of index k, $1 \leq k \leq nts[i,op]$, in the sequence $ats[i,op]$ of the ages of the transferred spare satellites, the failure $ft[i,op,k]$, $ft[i,op,k] \in \{0,1\}$, either of the transfer, or of the operational running of the associated satellite;
- for the third transition type after the third decision step:
  - for each orbital plane op, $1 \leq op \leq NOP$, the number $nfss[i,op]$, $0 \leq nfss[i,op] \leq nss[\langle i,3\rangle,op]$, of spare satellites that are present on this orbital plane and fail at instant i (during month i); more precisely, for each element of index k, $1 \leq k \leq nss[\langle i,3\rangle,op]$, in the sequence $ass[\langle i,3\rangle,op]$ of the ages of the present spare satellites, the failure $fs[i,op,k]$, $fs[i,op,k] \in \{0,1\}$, of the associated satellite;
  - for each orbital plane op, $1 \leq op \leq NOP$, the number $nfos[i,op]$, $0 \leq nfss[i,op] \leq nos[\langle i,3\rangle,op]$, of operational satellites that are present on this orbital plane and fail at instant i (during month i); more precisely, for each element of index k, $1 \leq k \leq nos[\langle i,3\rangle,op]$, in the sequence $aos[\langle i,3\rangle,op]$ of the ages of the present operational satellites, the failure $fo[i,op,k]$, $fo[i,op,k] \in \{0,1\}$, of the associated satellite.

The probability distributions that are associated with these random variables are the following:

- for the first transition type:
  - for the variable fl:
$$P(fl[i]=1) = PFL[ptl[\langle i,1\rangle,0]] \tag{6}$$
  - for the variable $nfrss$, if C_m^n is the number of combinations of n elements among m:
$$\begin{gathered}\forall n, 0 \leq n \leq NLS[ptl[\langle i,1\rangle,0]], \\ P(nfrss[i]=n) = C^n_{NLS[ptl[\langle i,1\rangle,0]]} \cdot \\ PFRSS^n \cdot (1-PFRSS)^{NLS[ptl[\langle i,1\rangle,0]]-n}\end{gathered} \tag{7}$$
- for the second transition type:
  - for the variables $nfros$:
$$\begin{gathered}\forall op, 1 \leq op \leq NOP, \\ \forall n, 0 \leq n \leq nts[i,op], \\ P(nfros[i,op]=n) = C^n_{nts[i,op]} \cdot \\ PFROS^n \cdot (1-PFROS)^{nts[i,op]-n}\end{gathered} \tag{8}$$
- for the third transition type:
  - for the variables $nfss$:
$$\begin{gathered}\forall op, 1 \leq op \leq NOP, \\ \forall n, 0 \leq n \leq nss[\langle i,2\rangle,op], \\ P(nfss[i,op]=n) = \textstyle\sum_{as \subseteq ass[\langle i,2\rangle,op],|as|=n} \\ ((\textstyle\prod_{a \in as} PFSS[a]) \cdot \\ (\textstyle\prod_{a \in (ass[\langle i,2\rangle,op]-as)} (1-PFSS[a])))\end{gathered} \tag{9}$$
  - for the variables $nfos$:
$$\begin{gathered}\forall op, 1 \leq op \leq NOP, \\ \forall n, 0 \leq n \leq nos[\langle i,2\rangle,op], \\ P(nfos[i,op]=n) = \textstyle\sum_{as \subseteq aos[\langle i,2\rangle,op],|as|=n} \\ ((\textstyle\prod_{a \in as} PFOS[a]) \cdot \\ (\textstyle\prod_{a \in (aos[\langle i,2\rangle,op]-as)} (1-PFOS[a])))\end{gathered} \tag{10}$$
  - for the variables fs, if $a[k]$ is the age of the element of index k in the sequence $ass[\langle i,2\rangle,op]$:
$$\begin{gathered}\forall op, 1 \leq op \leq NOP, \\ \forall k, 0 \leq k \leq nss[\langle i,2\rangle,op], \\ P(fs[i,op,k] = PFSS[a[k]])\end{gathered} \tag{11}$$
  - for the variables fo, if $a[k]$ is the age of the element of index k in the sequence $aos[\langle i,2\rangle,op]$:
$$\begin{gathered}\forall op, 1 \leq op \leq NOP, \\ \forall k, 0 \leq k \leq nos[\langle i,2\rangle,op], \\ P(fo[i,op,k] = PFOS[a[k]])\end{gathered} \tag{12}$$

These random variables allow us to write state transition equations:

- for the first transition type, changes affect, on the one hand, the number nss of spare satellites (and the associated sequence ass of ages) on the chosen launch orbital plane, on the other hand, the current launching plan and the constraints on its possible extensions (ptl and mtl) in case of launcher failure (increase in the minimum time before a launch of the same type); because they are obvious, but may be very cumbersome to write and to read, we do not provide the reader with the exact definition of the changes in ass, ptl, and mtl;

$$\begin{gathered}\forall op, 1 \leq op \leq NOP, op \neq lop[i], \\ nss[\langle i,2\rangle,op] = nss[\langle i,1\rangle,op]\end{gathered} \tag{13}$$

$$(lop[i] > 0) \Rightarrow \quad (nss[\langle i,2\rangle, lop[i]] = nss[\langle i,1\rangle, lop[i]] + (1 - fl[i]) \cdot (NLS[tl[i,0]] - nfrss[i])) \tag{14}$$

both equations (equations 13 and 14) express that the number of spare satellites does not change, except on the possible launch orbital plane, where it is increased by the number of satellites whose launch and running on the spare orbit succeed;

- for the second transition type, changes affect the number of spare and operational satellites (and the associated sequences ass and aos of ages) on all the orbital planes (due to transfers of spare satellites from the spare orbit to the operational one) ; we do not provide the reader with the exact definition of the changes in ass and aos;

$$\forall op, 1 \le op \le NOP, \quad nss[\langle i,3\rangle, op] = nss[\langle i,2\rangle, op] - nts[i,op] \tag{15}$$

$$\forall op, 1 \le op \le NOP, \quad nos[\langle i,3\rangle, op] = nos[\langle i,2\rangle, op] + nts[i,op] - nfros[i,op] \tag{16}$$

both equations (equations 15 and 16) express that, on each orbital plane, the number of spare satellites is decreased by the number of transferred satellites and that the number of operational satellites is increased by the number of transferred satellites whose running on the operational orbit succeeds;

- for the third transition type, changes affect, on the one hand, the number of spare and operational satellites (and the associated sequences of ages) on all the orbital planes (due to possible failures during month i and to the drifting of the spare satellites on their spare orbits), on the other hand, the launching plan and the constraints on its possible extensions (due to its one step extension); we do not provide the reader with the exact definition of the changes in ass and aos;

$$\forall op, 2 \le op \le NOP, \quad nss[\langle i+1,1\rangle, op] = nss[\langle i,3\rangle, op-1] - nfss[i, op-1] \tag{17}$$

$$nss[\langle i+1,1\rangle, 1] = nss[\langle i,3\rangle, NOP] - nfss[i, NOP] \tag{18}$$

$$\forall op, 1 \le op \le NOP, \quad nos[\langle i+1,1\rangle, op] = nos[\langle i,3\rangle, op] - nfos[i,op] \tag{19}$$

$$\forall i', 0 \le i' \le DH-1, \quad ptl[\langle i+1,1\rangle, i'] = ptl[\langle i,3\rangle, i'+1] \tag{20}$$

$$ptl[\langle i+1,1\rangle, DH] = etl[i] \tag{21}$$

$$\forall tl, 0 \le tl \le NTL, tl \ne etl[i], \quad mtl[\langle i+1,1\rangle, tl] = max(DH+1, mtl[\langle i,3\rangle, tl] - 1) \tag{22}$$

$$mtl[\langle i+1,1\rangle, etl[i]] = DH + MTL[etl[i]] \tag{23}$$

the first two equations (equations 17 and 18) express that, on each orbital plane, the number of spare satellites is equal to the number of spare satellites that were present on the previous orbital plane the previous month and did not fail during this month; the third one (equation 19) expresses that, on each orbital plane, the number of operational satellites is equal to the number of operational satellites that were present the previous month and did not fail during this month; the fourth one (equation 20) results from the shift of the launching plan, month after month; the fifth one (equation 21) results from the adding of the current decision at the end of the current launching plan; the last two ones (equations 22 and 23) result from the decreasing, month after month, for each type of launcher, of the minimum time before a launch of this type, except for the type that is associated with the current decision.

Combining these equations with the probability distributions that are associated with these random variables allow us to get straightforwardly conditional probability distributions on the state variables at the following step. For example, for the first transition type:

$$\forall op, 1 \le op \le NOP, \quad \forall n, n' \ge 0, n \ne n', \quad P(nss[\langle i,2\rangle, op] = n' \mid nss[\langle i,1\rangle, op] = n, lop[i] \ne op) = 0 \tag{24}$$

$$\forall op, 1 \le op \le NOP, \quad \forall n \ge 0, \quad P(nss[\langle i,2\rangle, op] = n \mid nss[\langle i,1\rangle, op] = n, lop[i] \ne op) = 1 \tag{25}$$

$$\forall op, 1 \le op \le NOP, \quad \forall n, n' \ge 0, n' < n, \quad P(nss[\langle i,2\rangle, op] = n' \mid nss[\langle i,1\rangle, op] = n, lop[i] = op) = 0 \tag{26}$$

$$\forall op, 1 \leq op \leq NOP, \tag{27}$$
$$\forall n, n' \geq 0, n + NLS[tl[i,0]] < n',$$
$$P(nss[\langle i,2\rangle, op] = n' \mid nss[\langle i,1\rangle, op] = n, lop[i] = op)$$
$$= 0$$

$$\forall op, 1 \leq op \leq NOP, \tag{28}$$
$$\forall n, n' \geq 0, n \leq n' \leq n + NLS[tl[i,0]],$$
$$P(nss[\langle i,2\rangle, op] = n' \mid nss[\langle i,1\rangle, op] = n, lop[i] = op)$$
$$= P(fl[i] = 0) \cdot$$
$$P(nfrss[i] = n - n' + NLS[tl[i,0]])$$

$$\forall op, 1 \leq op \leq NOP, \tag{29}$$
$$\forall n, 1 \leq n \leq NOP,$$
$$P(nss[\langle i,2\rangle, op] = n \mid nss[\langle i,1\rangle, op] = n, lop[i] = op)$$
$$= P(fl[i] = 1) +$$
$$P(fl[i] = 0) \cdot P(nfrss[i] = NLS[tl[i,0]])$$

The first two equations (equations 24 and 25) express that the number of spare satellites does not change on any orbital plane that is different from the chosen launch orbital plane. The other four (equations 26, 27, 28, and 29) are related to the chosen orbital plane: the first of them (equation 26) expresses that the number of spare satellites cannot decrease on this orbital plane; the second one (equation 27) expresses that it cannot increase of more than the number of launched satellites; the third one (equation 28) expresses that it can increase strictly only if the launch succeeds; the fourth one (equation 29) expresses that it can remain the same, either if the launch fails, or if running the launched satellites fails for all of them.

Local and global costs

We consider that the cost of a transition of the two first types is null, but that the cost of a transition of the third type is the sum of two elements:

- the cost of the launch that has been planned, which is itself the sum of the cost of the launcher and of the launched satellites; note that means that the launcher and the satellites must be paid as soon as they are planned;
- the cost of the complete or partial unavailability of the constellation during month i.

This cost is thus given by the formula:

$$CL[etl[i]] + NLS[etl[i]] \cdot CS + \tag{30}$$
$$CU[\{nos[\langle i+1,1\rangle, op] \mid 1 \leq op \leq NOP\}]$$

As always with costs, the global cost is the sum of the local costs. In case of an infinite temporal horizon, the discounting factor γ is applied each month.

Deployment, maintenance, and renewal problems

It may be profitable to distinguish three different planning problems, each of them using the same basic model:

- the *deployment* problem, where the initial state ($i = 0$) is made up of neither spare nor operational satellite and of an empty launching plan (the variables nss, nos, tl are all equal to 0, the variables ass and aos to empty, and the variables mtl to $DH+1$) and where the goal state is made up of NOS satellites on each orbital plane (the variables nos are all equal to NOS); in this problem, the temporal horizon is finite, but not bounded (we do not know exactly when the constellation will be fully deployed); the ages of the satellites are not very different; thus, it may be not necessary to take them into account in the state of the constellation; moreover, one can assume that each month before the full deployment of the constellation costs a given amount of money;
- the *maintenance* problem, where the objective is to maintain as best as possible the goal state of the previous problem, over a finite or infinite horizon; in this problem, the ages of the satellites may be very different from each other; thus, it may be necessary to take them into account in the state of the constellation; moreover, one can assume that each month of partial unavailability costs an amount of money that is a function to be precisely defined of the number of missing operational satellites on each orbital plane;
- the *renewal* problem, which can be seen as a variant of the previous problem, with this special feature that most of the satellites arrive nearly at the same time at the end of their operational life.

Numerical values

Here is a set of typical values for this problem:

- number of orbital planes:
  $NOP = 8$;
- number of operational satellites per orbital plane:
  $NOS = 4$;
- number of available types of launcher:
  $NTL = 2$;
- number of satellites that can be launched by a launcher:
  $NLS[1,2] = [4,2]$;
- minimum time between two launches of the same type (in months):
  $MTL[1,2] = [4,2]$;
- minimum time between two launches of the same type in case of failure of the first one (in months):
  $MTLF[1,2] = [8,4]$;
- decision horizon (length of the current launching plan, in months):
  $DH = 6$;
- probability of failure of a launch:
  $PFL[1,2] = [0.1, 0.05]$;

- probability of failure when running a satellite on a spare orbit:
  $PFRSS = 0.1$;
- probability of failure when transferring a satellite from a spare orbit to an operational one and running it on the operational orbit:
  $PFROS = 0.05$;
- probability of failure on a spare orbit in a month:
  $\forall a, a < 72, PFSS[a] = 0.01$
  $PFSS[72] = 1$
  (probability independent of the age until 6 years; end of life at 6 years);
- probability of failure on a operational orbit in a month:
  $\forall a, a < 72, PFOS[a] = 0.02$
  $PFOS[72] = 1$
  (the same thing as with the probability of failure on a spare orbit);
- cost of a launch:
  $CL[1, 2] = [0.5, 0.3]$;
- cost of a satellite:
  $CS = 0.1$;
- cost of any partial unavailability of the constellation (a complete availability is assumed to be required at any moment):
  $CU = 1$;
- cost discounting factor:
  $\gamma = 0.99$.

These values give us an idea of the dimension of the state and action spaces. If we assume that there are at most NOS satellites on the spare and on the operational orbits on each orbital plane, if we do not take into account the ages of the satellites that dramatically increase the dimension of the state space, but if we do not take into account various constraints and symmetries that can reduce the dimension of the state and action spaces:

- the dimension SD of the state space is equal to:

$$(NOS + 1)^{2 \cdot NOP} \cdot (NTL + 1)^{DH+1} \cdot \prod_{tl=0}^{NTL} MTL[tl]$$

- the dimension AD of the action space is equal to:

$$(NOP + 1) \cdot (NOS + 1)^{NOP} \cdot (NTL + 1)$$

With the numerical values, we gave as an example, we obtain the following values:

- $SD = 5^{16} \cdot 3^7 \cdot 4 \cdot 2 \simeq 2.67 \cdot 10^{15}$;
- $AD = 9 \cdot 5^8 \cdot 3 \simeq 1.05 \cdot 10^7$;

Whereas the number of decisions to consider in each state seems to be reasonable, the number of states to consider is huge and becomes astronomical if we want to introduce the age of each satellite.

Possible approaches

As we already said, this modelling does not prejudge the use of any specific solving approach. Among the numerous possible approaches, one can roughly distinguish between *knowledge-based*, *model-based*, and *simulation-based* approaches:

- *knowledge-based* approaches assume the existence of an *expert knowledge* about what is the best thing to do in any possible state of the system; in expert systems (Hayes-Roth 1992), this knowledge is organized into decision rules; in case-based reasoning systems (Jona & Kolodner 1992), it is organized into cases; we can however observe that constellations of satellites are new systems and that nobody has a great experience about their deployment and maintenance; in such a context, these approaches do not seem to be the most appropriate;
- *model-based* approaches assume the existence of a *model* of the system and of its dynamics, as the one we described in the previous section (model of the own dynamics of the system and of its dynamics in response to possible actions); among these approaches, one can further distinguish between *planning* approaches and *sequential decision making* approaches:
  - *planning* approaches globally aim at producing a plan of actions over a specified temporal horizon; for example, with classical Artificial Intelligence planning (Fikes & Nilsson 1971; Blum & Furst 1997), models are organized into models of actions (their conditions and their effects), actions are assumed to be deterministic, and a plan is searched for to go from an initial state of the system to a goal state which satisfies specified conditions; when an action fails or the state of the system differs from the one that was waited for, either a new plan is built, or the previous one is repaired; planning, execution, and replanning alternate; probabilistic planning (Kushmerick, Hanks, & Weld 1995) introduces non deterministic actions and slightly modifies the objective of the planning system: it is no more to produce a plan that certainly allows the system to go from the initial state to a goal state; it is now to produce a plan such that the probability of going from the initial state to a goal state is the highest;
  - *sequential decision making* approaches aim at determining, at each instant when an action is possible, the best action to perform; they can be seen as planning approaches where the temporal decision horizon is limited to one action; Markov Decision Processes (MDP) are a standard model for sequential decision making; as we saw, MDP models are organized into models of states, actions, transitions, and gains or costs; the standard objective is to determine what is called an optimal policy, that is a function that associates with any possible state of the system and any instant, the action that maximizes the expected gain (or minimizes the expected cost) over the remaining temporal horizon; *dynamic programming*, *value iteration*, and *policy iteration* are the most known algorithms for computing off-line, that is before execution, an optimal policy;

in the context of the deployment and of the maintenance of a constellation of satellites, one can observe that uncertainty is high and that the next action to perform can be decided at each instant; under these conditions, building a plan over a large horizon seems to be neither useful nor optimal: it would have a high probability to become quickly obsolete; a *sequential decision making* approach seems to be the most appropriate; note that this approach does not impose that the optimal or approximately optimal policy be entirely computed *off-line*: the best action (or a good action) to perform at each instant can be at least partially *on-line* computed, according to the actual state of the system at this instant;

- *simulation-based* approaches use the experience that is provided by either the real life of the system or by a simulation of this life to learn an approximately optimal policy; the objective is thus similar to the one of the model-based sequential decision making approaches; moreover, simulation-based approaches may use or learn models that are similar to the ones that are used by the model-based sequential decision making approaches; thus, the frontier between both these kinds of approach is rather fuzzy; simulation-based approaches, such as *reinforcement learning* (Sutton & Barto 1998) or *neuro-dynamic programming* (Bertsekas & Tsitsiklis 1996), can be seen in fact as sorts of model-based sequential decision-making approaches; but, some simulation-based approaches, such as *stochastic optimization* (Fu 2001) do not use any markovian model and aim directly at optimizing the global cost or gain function, which is a stochastic function of the policy; in the context of the deployment and of the maintenance of a constellation of satellites, using the real life of the system to learn a good quality policy is out of question, because real experiences are too rare and too costly, but simulations can be extensively used at the design phase of the constellation, months before its actual launch; thus, these approaches may be really relevant.

In (Garcia *et al.* 2001), the reader can find a comparison of the results that have been obtained on the maintenance problem, with two simulation-based approaches: an *approximate policy iteration* approach and a *stochastic optimization* approach. The best results have been obtained with the second one. But, we are convinced that these results are still preliminary and can be greatly improved, for example by a combination of *off-line* and *on-line* computations. We are also convinced that the main obstacle to overcome is still the astronomical number of possible states to deal with, especially when we need to take into account the ages of the satellites to determine a good quality maintenance policy.

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